RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SIXTH SEMESTER EXAMINATION, MAY 2025

THIRD YEAR (BATCH 2022-25)

Date : 05/05/2025 **MATHEMATICS (HONOURS)**

Time : 11.00 am – 1.00 pm **Paper : CC 13** Full Marks : 50

All the symbols have their usual meaning.

***Answer ALL the questions, maximum you can obtain is 50*. You may use a non-programmable calculator.**

1. For a function *f*, the forward divided differences are

$x_0 = 0.0$	$f[x_0] = ?$		
		$f[x_0, x_1] = ?$	
$x_1 = 0.4$	$f[x_1] = ?$		$f[x_0, x_1, x_2] = \frac{50}{7}$
		$f[x_1, x_2] = 10$	
$x_2 = 0.7$	$f[x_2] = 6$		

Determine the missing entries of the table. [5]

2. a) Establish Newton's divided difference interpolation formula.
   b) Suppose $f(x) = \sin x$ is approximated by an interpolating polynomial $p(x)$, of degree equal to a two digit number that you prefer, in the interval [0,1]. Estimate $|f(x) - p(x)|$ in [0,1]. [5+2]

3. Establish Lagrange's differentiation formula. [7]

4. State and prove the convergence criteria for fixed point iteration method. [6]

5. Find y(4.4) by Euler's modified method taking h = 0.2 from the differential equation $\frac{dy}{dx} = \frac{2 - y^2}{5x}, y = 1$ when $x = 4$. [5]

6. Establish composite Trapezoidal rule for finding $\int_{x_0}^{x_n} f(x)\,dx$. State geometrical interpretation of Trapezoidal rule. [4+1]

7. Suppose $x_0, x_1, x_2 ..... x_n$ be *n* distinct real numbers in the interval [*a*,*b*] and $f \in C^{n+1}[a,b]$. Then show that for each $x \in [a,b]$, a number $\xi(x)$ (generally unknown) between $x_0, x_1, x_2, ..., x_n$ and hence in (*a*,*b*) exits, so that $f(x) = p_n(x) + \frac{f^{n+1}(\xi(x))}{(n+1)!}(x - x_0)(x - x_1).....(x - x_n)$ where $p_n(x)$ is the Lagrange's interpolating polynomial. [5]

8. Which of the following three functions will you choose to approximate the root of $x^3 - x = 0.1$ in the interval [1,2] and why? The three functions are :-

$\phi_1(x) = (x + 0.1)^{\frac{1}{3}}$, $\phi_2(x) = (x^3 - 0.1)$, $\phi_3(x) = \dfrac{0.1}{x^2 - 1}$. [7]

9. Describe Newton-Raphson method. State and prove its sufficient condition for convergence. [4+1+5]

10. Define : [1×3]
(i) Significant figure
(ii) Round off error
(iii) Truncation error

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